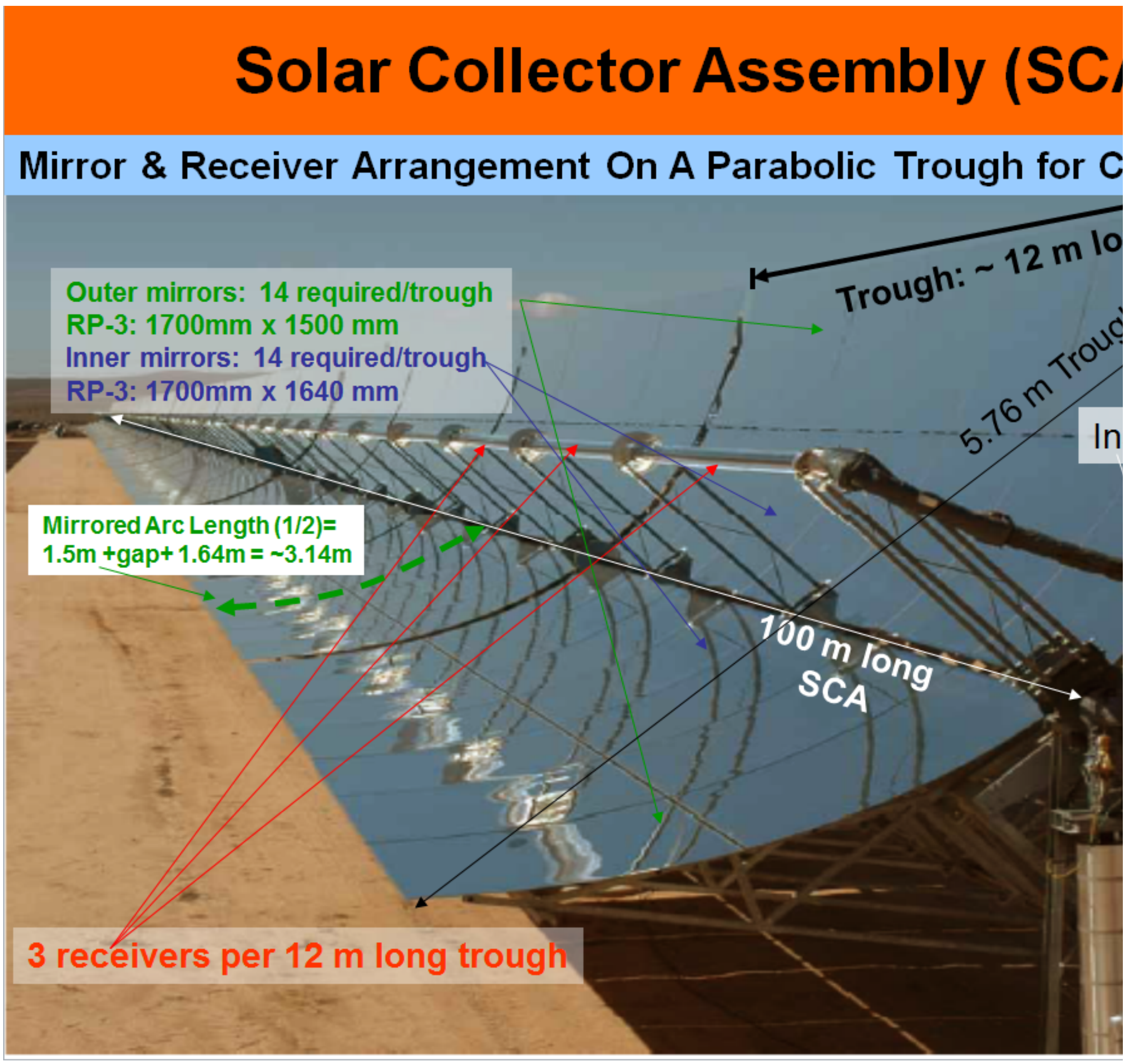
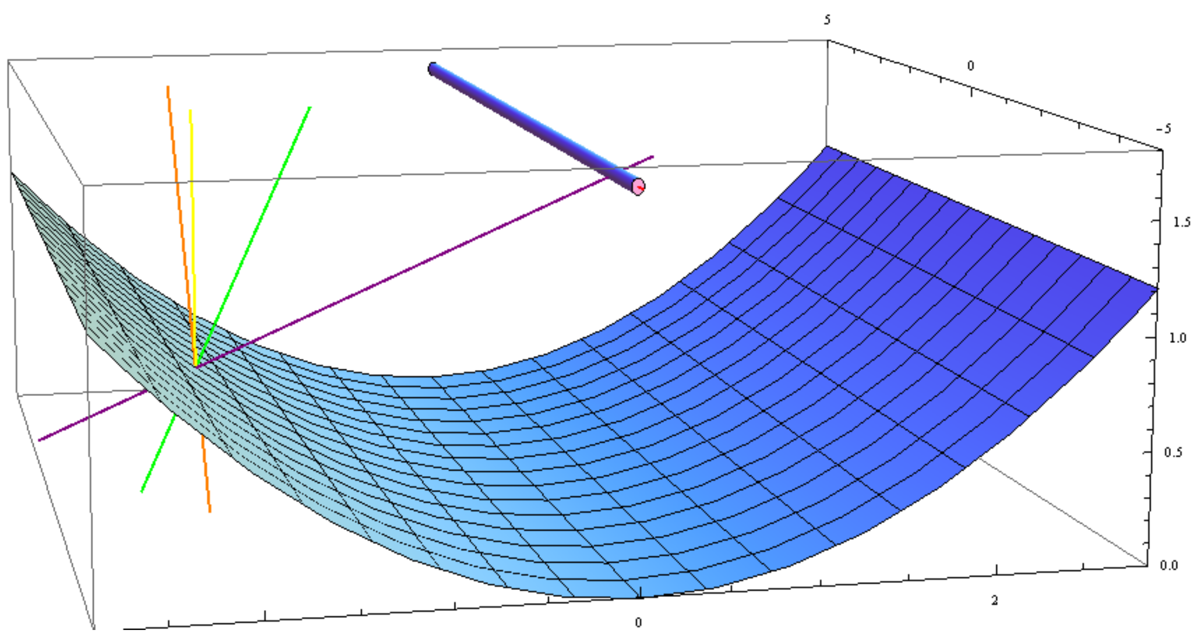
In[1]:=

Incoming solar ray traced to the receiver of a Solar Collector Assembly or SCA

In[2]:=

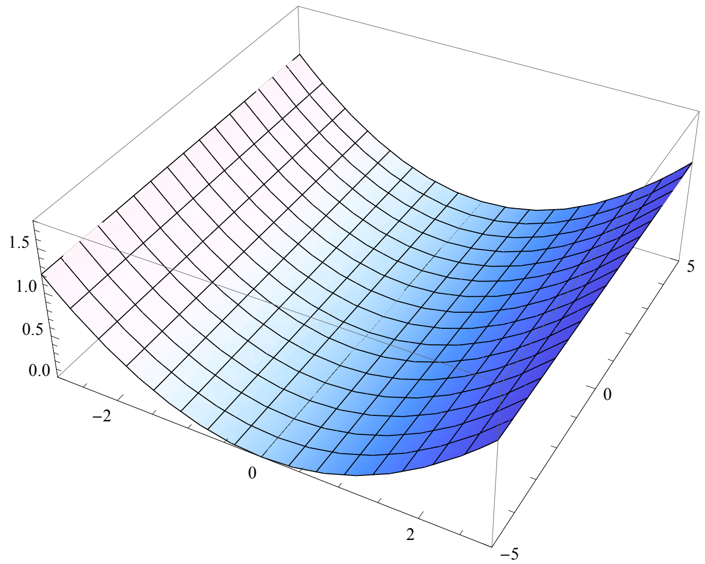


(* The objective is to show the incoming ray from the sun [orange color] reflected off the mirror trough [as the purple ray or line] at a point showing the local normal [the green line] and a line parallel to the z axis [the yellow line]. The long cylinder at the focal line is the receiver and as shown the receiver intercepts the purple ray.*)



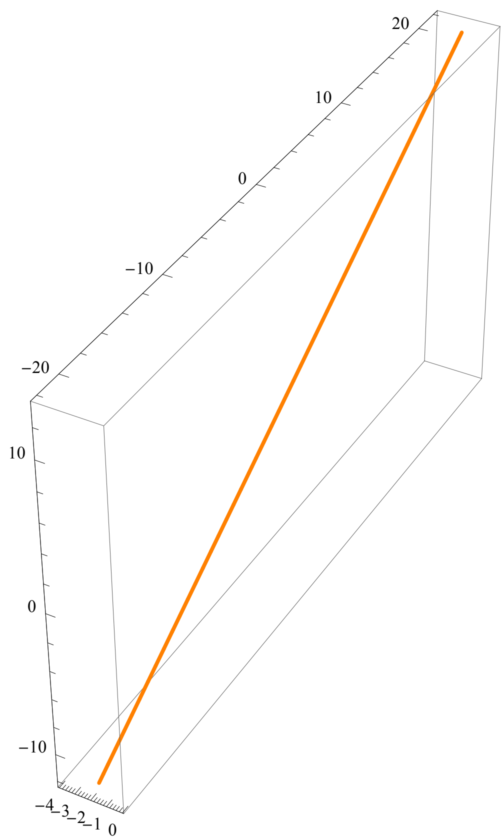
In[7]:=
```
f = 1.71;
(*Trough=Plot3D[x^2/(4f),{x,-5.76/2,5.76/2},{y,-5.76/2,5.76/2},
  PlotRange→{{-5.76/2,5.76/2},{-5.76/2,5.76/2},{0,1.8}}]*)
f = 1.71;
Trough = Plot3D[x^2/(4 f), {x, -5.76/2, 5.76/2},
  {y, -5, 5}, PlotRange → {{-5.76/2, 5.76/2}, {-5, 5}, {0, 1.8}}]
```

Out[9]=



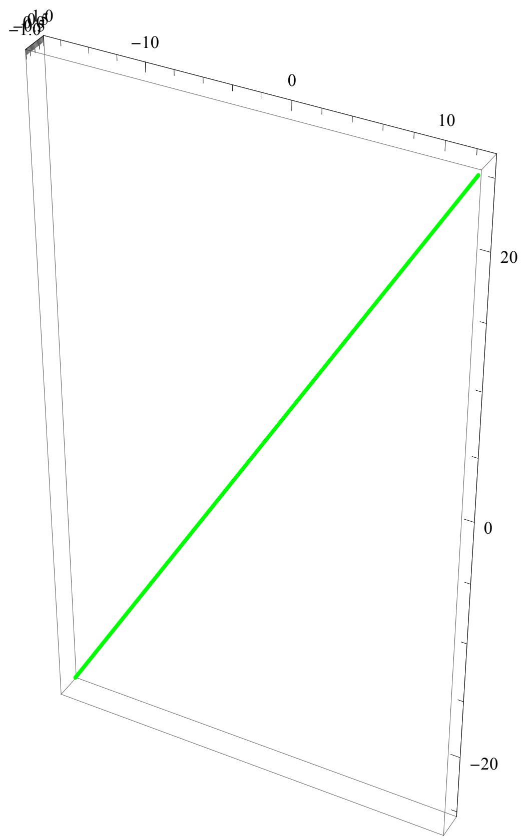
In[10]:=
```
Clear[x, y, f, θ]
f = 1.71; θ = 60; x = -2; y = 0;
r = {x, y, x^2/(4 f)} + t {0, -Sin[θ Degree], -Cos[θ Degree]};
(*of the form [point + vector lengthened by t] *)
IncomingRay =
 ParametricPlot3D[r, {t, -25, 25}, PlotStyle → Directive[Thick, Orange]]
```

Out[13]=



In[14]:=
```
Clear[f, x0, α, θ, β, m, y0, x , y, z]
f = 1.71;
x = -2; y = 0; z = x^2/(4 f);
(*line is given by gradient of equation for plane: plane =p(x,y,z)=z-x^2/(4 f) ,
gradient={-x/(2 f),0,1}*)
normal = {x, y, z} + t {-x/(2 f), 0, 1};
NormalLine = ParametricPlot3D[normal, {t, -25, 25}, PlotStyle → {Thick, Green}]
```

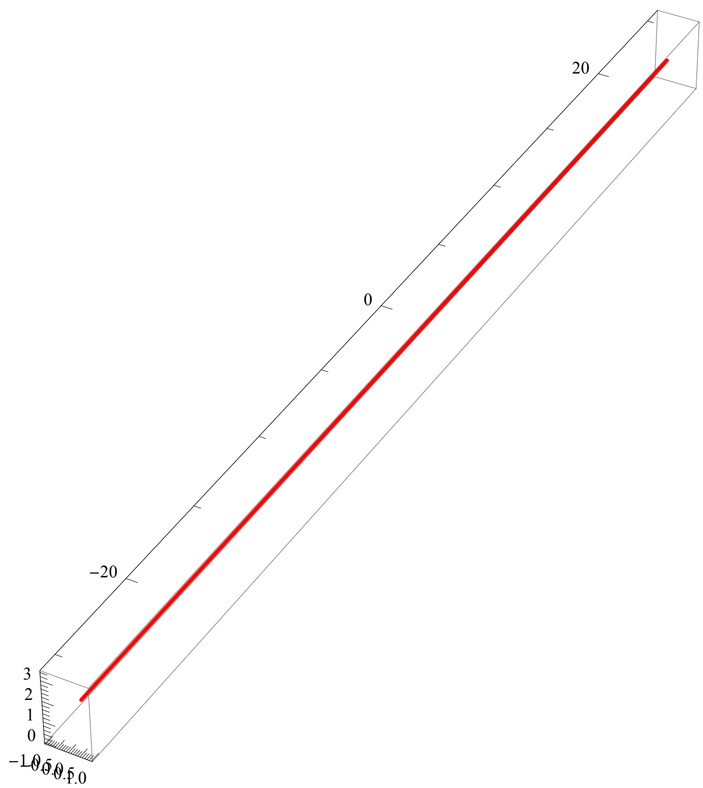
Out[18]=



In[19]:=
```
Clear[x, y, f, θ]
f = 1.71;  x = 0; y = 0; z = f;
fl = {x, y, z} + t {0, 1, 0}
FocalLine = ParametricPlot3D[fl, {t, -25, 25}, PlotStyle → Directive[Thick, Red]]
```

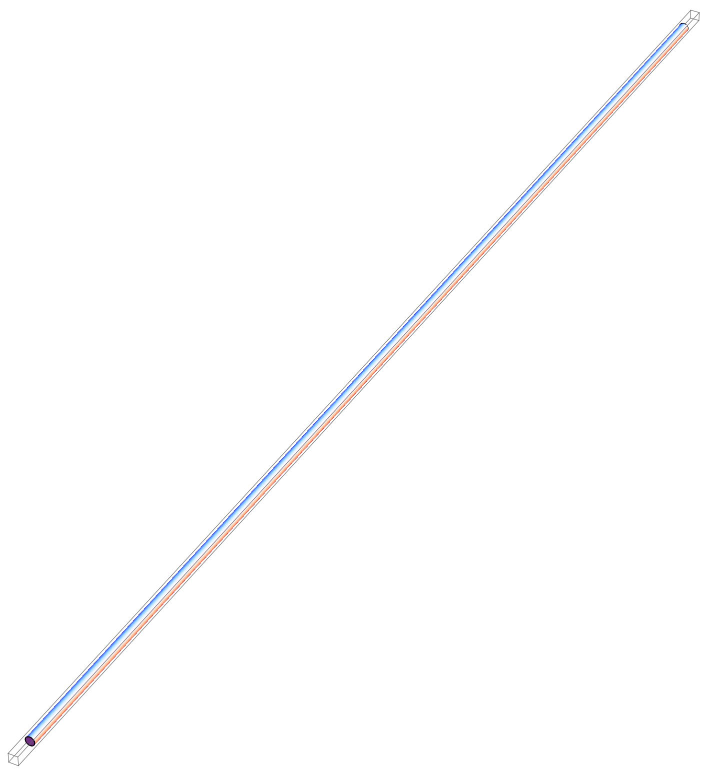
Out[21]= `{0, t, 1.71}`

Out[22]=



In[23]:=
```
Clear[x, y, f, θ]
f = 1.71;  x = 0;  y = 0; z = f;
CylinderLine =
 Graphics3D[Cylinder[{{x, y, z} - 4.8 {0, 1, 0}, {x, y, z} + 4.8 {0, 1, 0}}, 70/1000 1/2]]
```

Out[25]=



In[26]:=
```
(*See "H:\\MATHEMATICA 2009\\Equation of Plane.nb" for understanding
 the solution below in which we solve for z in the final equation,
two methods are shown, one uses vectors other uses determinante,
both results were exactly the same and
 plots of plane for both solution cases are shown.
  See Schaum's Advanced Calculus, p. 149, #28 for detailed explanation*)
Clear[x, y, z, f, θ, x0, y0]
vectorxyz = {x, y, z};
vectorP = {x0, y0, x0^2/(4 f)};
vectorS = vectorP + {0, -Sin[θ Degree], -Cos[θ Degree]};
(*negativevectorN=vectorP+{-x/(2f),0,1}*)
negativevectorN = vectorP + {-x0/(2 f), 0, 1};
```

In[31]:=
```
(*(vectorxyz-vectorP).Cross[(vectorS-vectorP),(negativevectorN-vectorS)]*)
```

In[32]:=
```
zz = z /. Solve[(vectorxyz - vectorP).
    Cross[(vectorS - vectorP), (negativevectorN - vectorS)] == 0, z]
```

Out[32]= $\left\{\frac{-8 f^2 x + 8 f^2 x0 + x0^3 + 4 f x0 y \text{Cot}[° \theta] - 4 f x0 y0 \text{Cot}[° \theta]}{4 f x0}\right\}$

In[33]:=
```
(*Clear[x,y,z,f,θ,x0,y0]
 vectorxyz={x,y,z}
  vectorP={x0,y0,x0^2/(4 f)}
   vectorS=vectorP+{0,-Sin[θ Degree], -Cos[θ Degree]}
     (*negativevectorN=vectorP+{-x/(2f),0,1}*)
    negativevectorN=vectorP+{-x0/(2f),0,1}*)
```

In[34]:=
```
(*MatrixForm[Array[a,{3,3}]]*)
```

In[35]:= $\begin{pmatrix} x - x0 & y - y0 & z - \frac{x0^2}{4 f} \\ x0 - x0 & y0 - (y0 - \text{Sin}[° \theta]) & \frac{x0^2}{4 f} - \left(\frac{x0^2}{4 f} - \text{Cos}[° \theta]\right) \\ x0 - \frac{x0}{2 f} - x0 & y0 - y0 & 1 + \frac{x0^2}{4 f} - \frac{x0^2}{4 f} \end{pmatrix}$

Out[35]= $\left\{\left\{x - x0,\ y - y0,\ -\frac{x0^2}{4 f} + z\right\},\ \{0,\ \text{Sin}[° \theta],\ \text{Cos}[° \theta]\},\ \left\{-\frac{x0}{2 f},\ 0,\ 1\right\}\right\}$

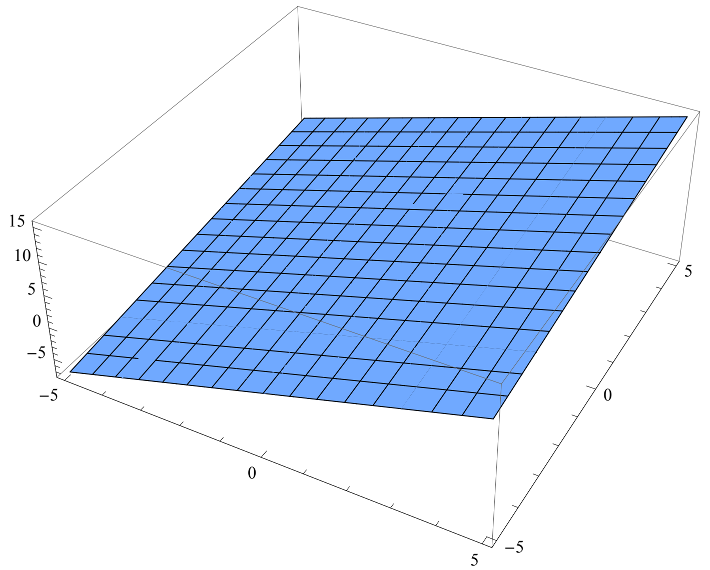
In[36]:=
```
(*Clear[x,y,z,f,θ,x0,y0]*)
```

$$(*\text{Det}\left[\begin{pmatrix} x-x0 & y-y0 & z-\frac{x0^2}{4 f} \\ x0-x0 & y0-(y0-\text{Sin}[° \theta]) & \frac{x0^2}{4 f}-\left(\frac{x0^2}{4 f}-\text{Cos}[° \theta]\right) \\ x0-\frac{x0}{2 f}-x0 & y0-y0 & 1+\frac{x0^2}{4 f}-\frac{x0^2}{4 f} \end{pmatrix}\right]*)$$

In[37]:= $(*\text{Solve}\left[-\frac{x0\ y\ \text{Cos}[° \theta]}{2 f}+\frac{x0\ y0\ \text{Cos}[° \theta]}{2 f}+x\ \text{Sin}[° \theta]-x0\ \text{Sin}[° \theta]-\frac{x0^3\ \text{Sin}[° \theta]}{8 f^2}+\frac{x0\ z\ \text{Sin}[° \theta]}{2 f}==0,z\right]*)$

In[38]:=
```
Clear[x, y, z, f, θ, x0, y0]
f = 1.71; θ = 60; x0 = -2; y0 = 0;
PlaneEquation = Plot3D[zz, {x, -5, 5}, {y, -5, 5}]
```

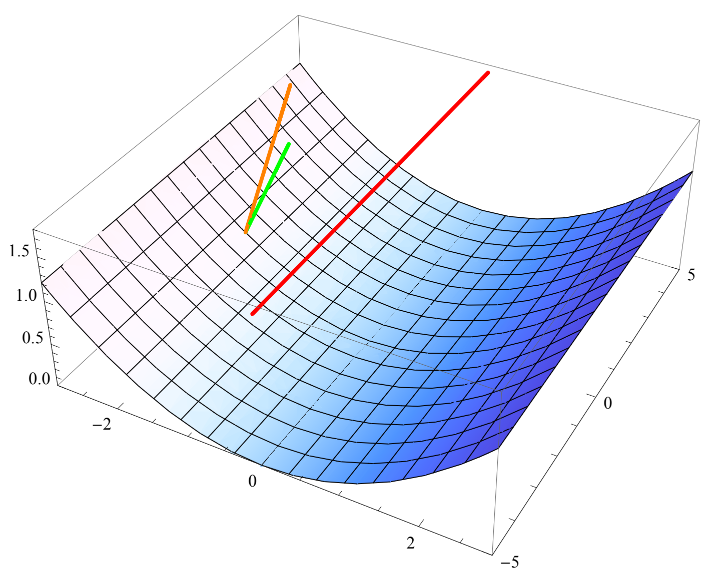
Out[40]=



In[41]:=
```
(*Clear[x,y,z,f,θ,x0,y0]
 f=1.71; θ=60; x0=-2; y0=0;
PlaneEquationDetWay=
 Plot3D[1/(4 f x0) Csc[° θ] (4 f x0 y Cos[° θ]-4 f x0 y0 Cos[° θ]-8 f^2 x Sin[° θ]+
     8 f^2 x0 Sin[° θ]+x0^3 Sin[° θ]),{x,-5,5},{y,0,10}]*)
```

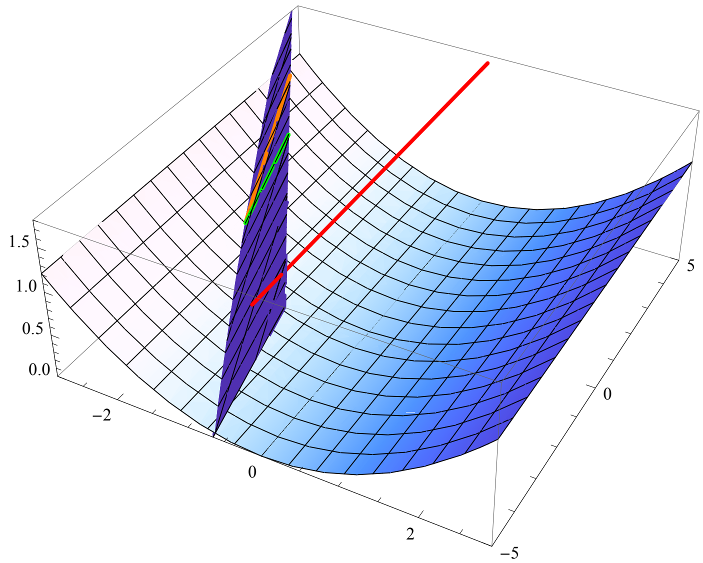
In[42]:=
```
Show[Trough, IncomingRay, FocalLine, NormalLine]
```

Out[42]=

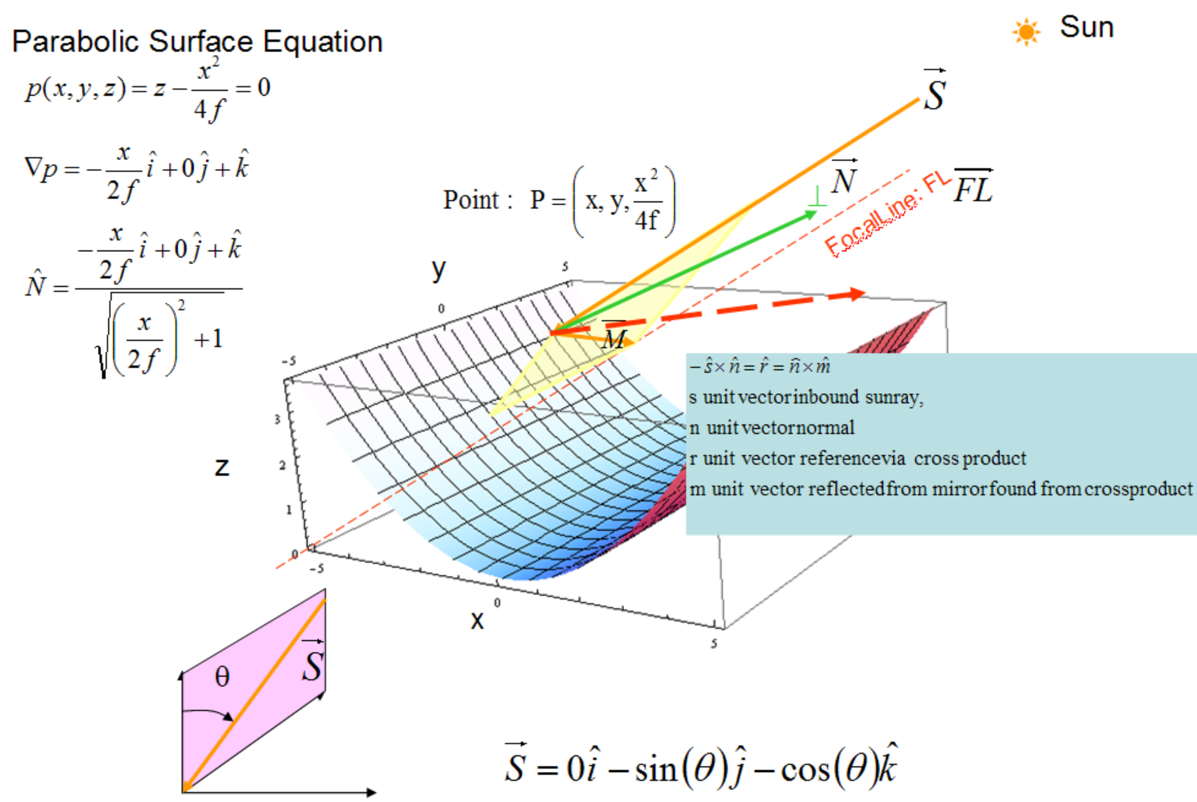


In[43]:=
```
Show[Trough, IncomingRay, FocalLine, NormalLine, PlaneEquation]
```

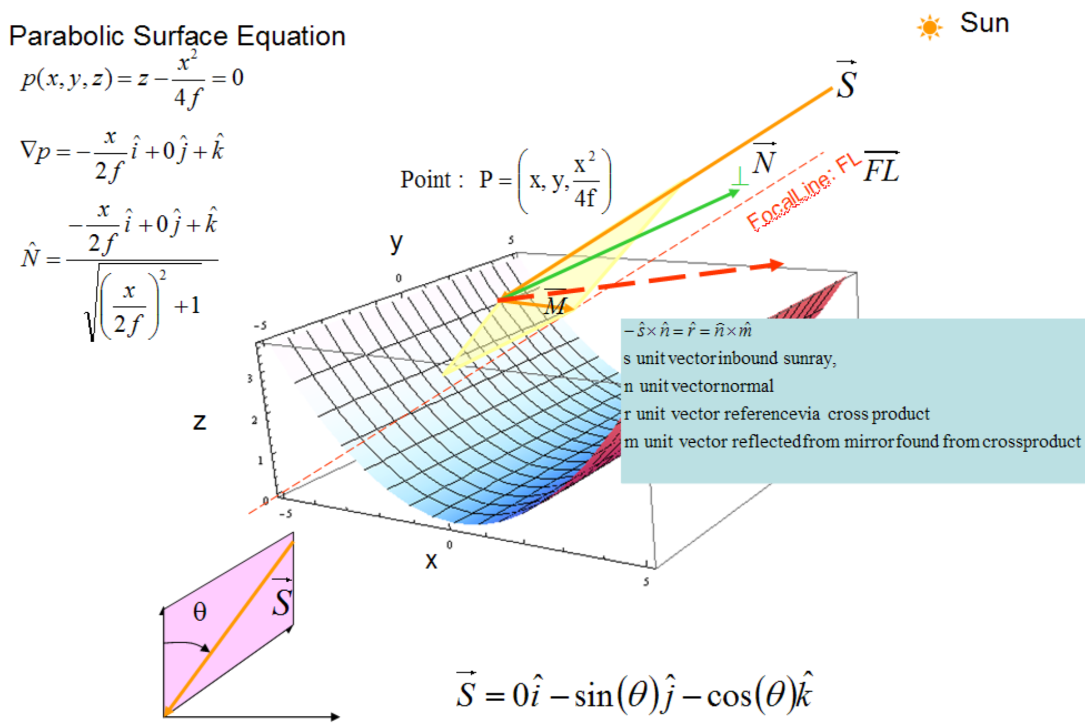
Out[43]=



In[44]:=



Out[44]=



In[45]:=
```
Clear[θ, x, y, z, f, nx, ny, nz, mx, my, mz, rx, ry, rz, sx, sy, sz];
Cross[{nx, ny, nz}, {mx, my, mz}]
```

Out[46]= {mz ny − my nz, −mz nx + mx nz, my nx − mx ny}

In[47]:=
```
(* n, r, m refer to normal,
reference(from cross) and mirror (i.e., reflected from mirror,respectively
    the -(nx sx +ny sy+nz sz) is to take care of the negative direction of S *)
Clear[nx, ny, nz, mx, my, mz, sx, sy, sz]
(*the -(nx sx +ny sy+nz sz) is to take care of the negative direction of S *)
Solve[{mz ny - my nz == rx, -mz nx + mx nz == ry, my nx - mx ny == rz,
  - (nx sx + ny sy + nz sz) == nx mx + ny my + nz mz, mx rx + my ry + mz rz == 0,
  nx rx + ny ry + nz rz == 0, sx rx + sy ry + sz rz == 0}, {mx, my, mz}]
```

Out[48]= { }

In[49]:=
```
Clear[θ, x, y, z, f, nx, ny, nz, mx, my, mz, rx, ry, rz]
normalvectorS = {0, -Sin[θ Degree], -Cos[θ Degree]};
normalvectorN = {-x/(2 f), 0, 1} / Sqrt[(-x/(2 f))^2 + 1];
{sx, sy, sz} = {0, -Sin[θ Degree], -Cos[θ Degree]};
{nx, ny, nz} = {-x/(2 f), 0, 1} / Sqrt[(-x/(2 f))^2 + 1];
{rx, ry, rz} = Cross[-normalvectorS, normalvectorN];
```

In[55]:=
```
Clear[mx, my, mz]
{mx, my, mz} =
 {(-nx ny nz rx + nx^2 nz ry - nx^2 ny rz - ny^3 rz - ny nz^2 rz - nx^4 sx - nx^2 ny^2 sx - nx^3 ny sy -
     nx ny^3 sy - nx^3 nz sz - nx ny^2 nz sz) / ((nx^2 + ny^2) (nx^2 + ny^2 + nz^2)), 1/(nx^2 + ny^2)
   (nx rz - nx ny sx - ny^2 sy - ny nz sz - (ny nz (ny rx - nx ry - nx nz sx - ny nz sy - nz^2 sz))/(nx^2 + ny^2 + nz^2)),
   (ny rx - nx ry - nx nz sx - ny nz sy - nz^2 sz)/(nx^2 + ny^2 + nz^2)}
```

Out[56]=
$$\left\{\frac{4 f^2 \left(1+\frac{x^2}{4 f^2}\right)\left(-\frac{x^3 \mathrm{Cos}[^\circ\theta]}{8 f^3\left(1+\frac{x^2}{4 f^2}\right)^2}-\frac{x^3\sqrt{\frac{4 f^2+x^2}{f^2}}\,\mathrm{Cos}[^\circ\theta]}{4 f\,(4 f^2+x^2)\left(1+\frac{x^2}{4 f^2}\right)^{3/2}}\right)}{x^2\left(\frac{1}{1+\frac{x^2}{4 f^2}}+\frac{x^2}{4 f^2\left(1+\frac{x^2}{4 f^2}\right)}\right)},\ -\mathrm{Sin}[^\circ\theta],\ \frac{\frac{\mathrm{Cos}[^\circ\theta]}{1+\frac{x^2}{4 f^2}}-\frac{x^2\sqrt{\frac{4 f^2+x^2}{f^2}}\,\mathrm{Cos}[^\circ\theta]}{2\,(4 f^2+x^2)\sqrt{1+\frac{x^2}{4 f^2}}}}{\frac{1}{1+\frac{x^2}{4 f^2}}+\frac{x^2}{4 f^2\left(1+\frac{x^2}{4 f^2}\right)}}\right\}$$

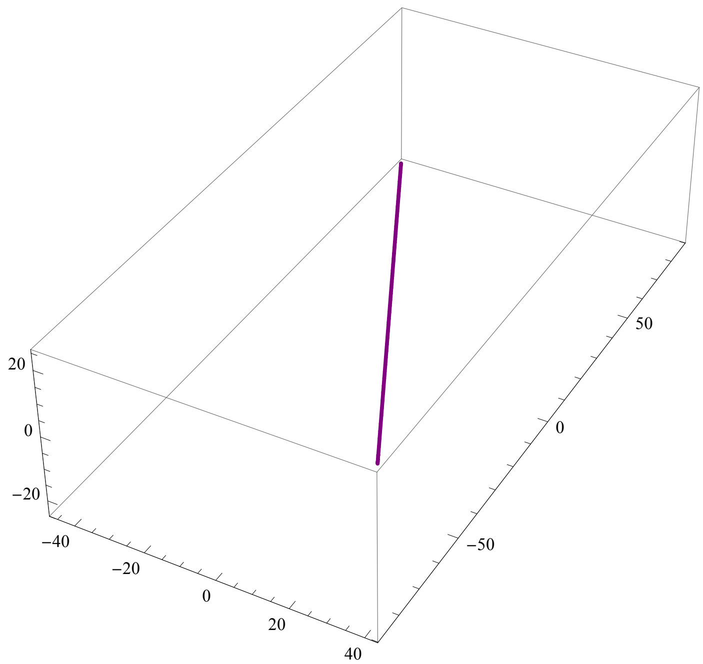
In[57]:=
```
f = 1.71; θ = 60;
x = -2; y = 0; z = x^2/(4 f);
{mx, my, mz}
```

Out[59]=
$$\left\{0.435769,\ -\frac{\sqrt{3}}{2},\ 0.245164\right\}$$

In[60]:=

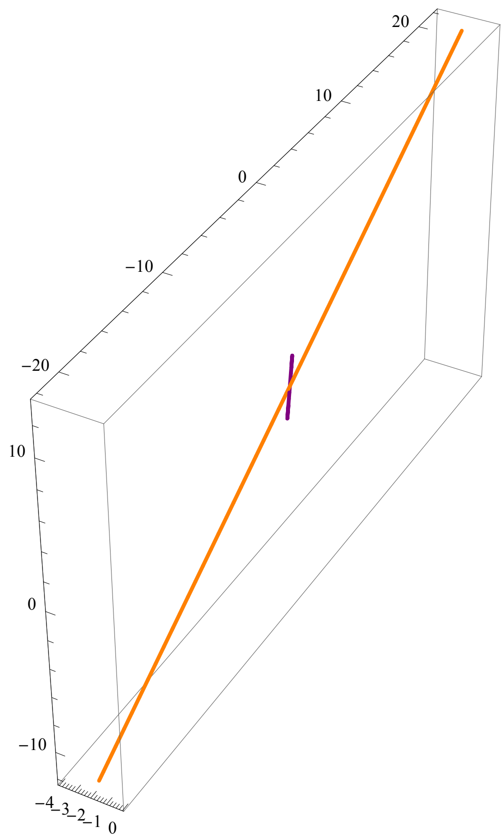
```
f = 1.71;
x = -2; y = 0; z = x^2/(4 f);
(*line is given by gradient of equation for plane: plane =p(x,y,z)=z-x^2/(4 f) ,
gradient={-x/(2 f),0,1}*)

reflected = {x, y, z} + t {mx, my, mz};
ReflectedLine =
 ParametricPlot3D[reflected, {t, -100, 100}, PlotStyle → {Thick, Purple}]
```

Out[63]=



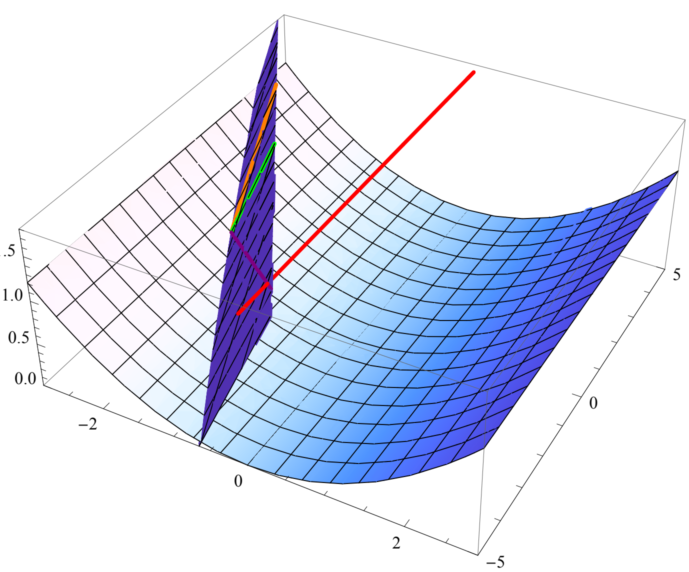
In[64]:=
```
Show[IncomingRay, ReflectedLine]
```

Out[64]=



In[65]:=
```
Show[Trough, IncomingRay, FocalLine, NormalLine, PlaneEquation, ReflectedLine]
```

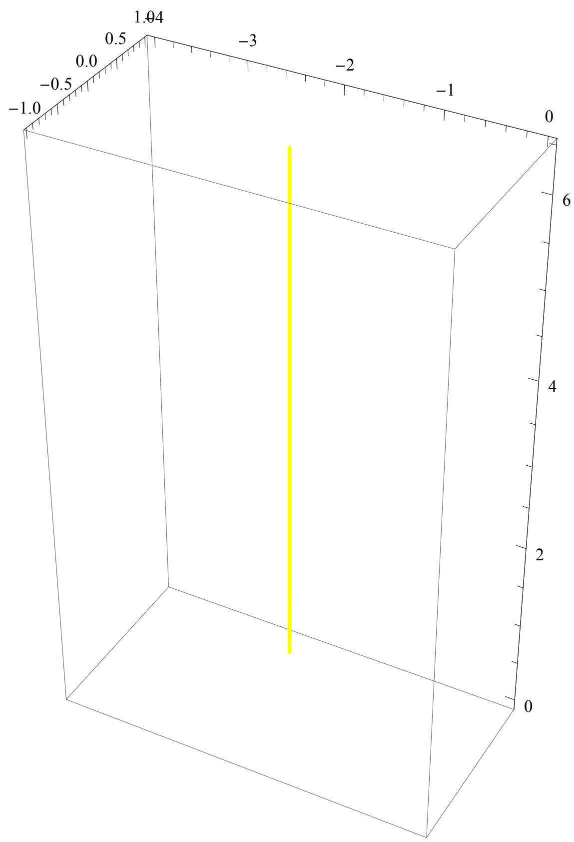
Out[65]=



In[66]:=

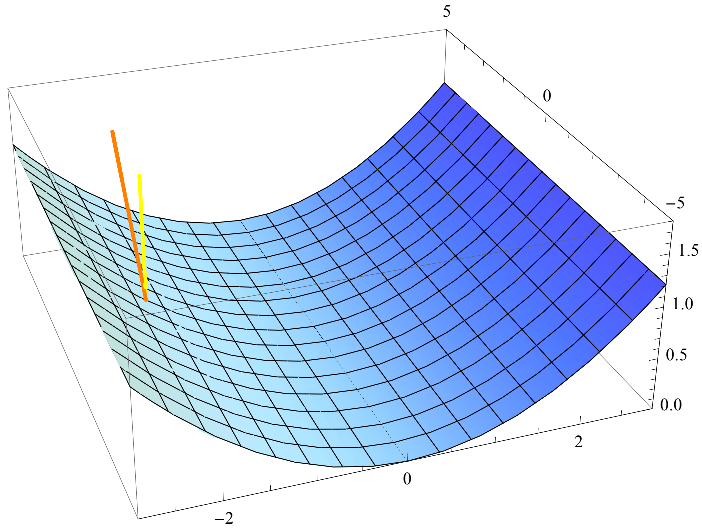
```
f = 1.71;
x = -2; y = 0; z = x^2/(4 f);
(*line is given by gradient of equation for plane: plane =p(x,y,z)=z-x^2/(4 f) ,
gradient={-x/(2 f),0,1}*)

paralleltoz = {x, y, z} + t {0, 0, z};
ParallelToZLine =
 ParametricPlot3D[paralleltoz, {t, 0, 10}, PlotStyle → {Thick, Yellow}]
```

Out[69]=



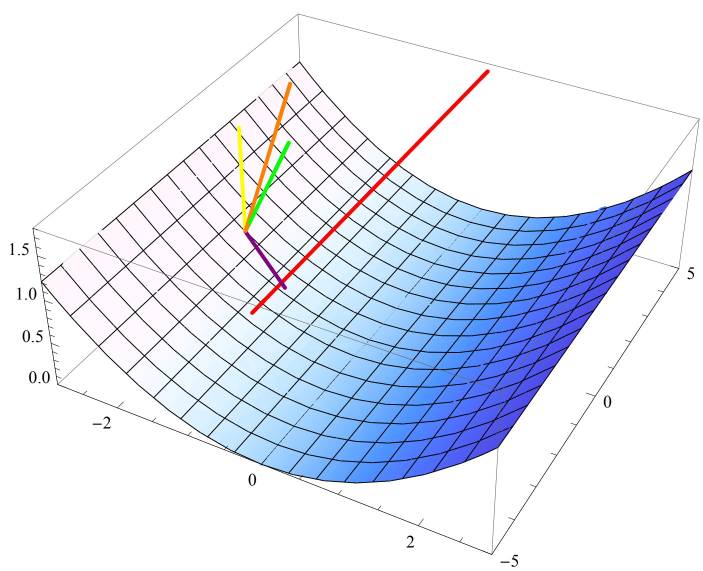
In[70]:=
```
Show[Trough, IncomingRay, ParallelToZLine]
```

Out[70]=



In[71]:=
```
Show[Trough, IncomingRay, FocalLine, ParallelToZLine, NormalLine, ReflectedLine]
```

Out[71]=



In[72]:=
```
Show[Trough, IncomingRay, FocalLine,
  ParallelToZLine, NormalLine, CylinderLine, ReflectedLine]
```

Out[72]=